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SOME THEORETICAL CONSIDERATIONS OF LONGITUDINAL STABILITY IN POWER-ON FLIGHT WITH SPECIAL REFERENCE TO WIND-TUNNEL TESTING

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SOME THEORETICAL CONSIDERATIONS OF LONGITUDINAL STABILITY IN POWER-ON FLIGHT WITH SPECIAL REFERENCE TO WIND-TUNNEL TESTING

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SUMMARY

Some problems relating to longitudinal stability in power-on flight are considered. A derivation is included which shows that, under certain conditions, the rate of change of the pitching-moment coefficient with lift coefficient as obtained in wind-tunnel tests simulating constant-power operation is directly proportional to one of the indices of stability commonly associated with flight analysis, the slope of the curve relating the elevator angle for trim and lift coefficient (or velocity). The necessity of analyzing power-on wind-tunnel data for trim conditions is emphasized and a method is provided for converting data obtained from constant-thrust tests to simulated constant-throttle flight conditions. It is demonstrated how a downward tail load required to trim an airplane results in decreased stability in power-on flight and why a longitudinal center-of-gravity movement is likely to affect the stability characteristics less in power-on flight than in power-off flight.

INTRODUCTION

The effect of running propellers on the longitudinal-stability characteristics of airplanes has been appreciated for many years. The increased use of powered models for wind-tunnel testing has greatly increased the amount of emperical information on the subject. The evaluation of wind-tunnel data secured with a power model, however, demands a greater appreciation of trim conditions than the evaluation of conventional power-off data. Data obtained from tests made with the propeller thrust held constant (reference 1), for example, consequently require an interpretation different from data obtained either from tests in

which the propeller thrust is permitted to vary or from tests in which the propeller is absent.

The purpose of the present paper is to correlate the different test procedures used in testing a model equipped with running propellers and to establish the significance of the data obtained for the determination of the longitudinal-stability characteristics. The slopes of the wind-tunnel pitching-moment curves are correlated with the index of stick-fixed stability commonly used in free-flight test work - the variation of elevator angle for trim with speed or lift coefficient. It is believed that a demonstration of the quantitative relationships existing between these different indices of stability would aid in the correlation of flight and wind-tunnel tests. The paper also considers the magnitude of the changes in the stick-fixed power-on stability of an airplane resulting from a change in center-of-gravity position and the different tail loads necessary for trim.

SYMBOLS AND FORMULAS

C_L lift coefficient

C_{L_t} lift coefficient of horizontal tail

C_D drag coefficient

C_{X_w} longitudinal force coefficient of wing

C_{m_o} pitching-moment coefficient of wing-fuselage combination about aerodynamic center

C_m pitching-moment coefficient of airplane excluding propeller-thrust component

C_M resultant pitching-moment coefficient (includes propeller-thrust component

c mean aerodynamic chord (M.A.C.)

c_e mean elevator chord

C_g ratio of distance of center of gravity back of leading edge of mean aerodynamic chord to mean aerodynamic chord

L-309

C_a ratio of distance of aerodynamic center of wing back of leading edge of mean aerodynamic chord to mean aerodynamic chord

z distance of mean aerodynamic chord below center of gravity

h distance of thrust axis below center of gravity

l'_1 distance from center of gravity to hinge line of horizontal tail

l_2 distance from center of gravity to plane of propeller disk

S_w wing area

S_t horizontal tail area

S_e elevator area

S_p propeller-disk area $(\pi D^2/4)$

D propeller diameter

n propeller rotational speed

V velocity of flight

V_s slipstream velocity

J advance-diameter ratio (V/nD)

K function of J and propeller-blade angle for an inclined propeller (reference 2)

T thrust

α angle of attack

γ flight-path angle

θ angle of airplane to horizontal

α_t angle of attack of horizontal tail

ϵ_w downwash angle at tail due to wing

ϵ_p downwash angle at tail due to propeller

i_t initial horizontal tail setting

δ_e elevator angle

W weight (mg)

m mass of the airplane

g gravity

k mechanical advantage of elevator-control system

k_Y radius of gyration about Y axis

P_D component of thrust coefficient along X axis during motion $\left[(T_c' + \frac{1}{2} T_c'_u) \cos \alpha\right]$

P_L component of thrust coefficient along Z axis during motion $\left[(T_c' + \frac{1}{2} T_c'_u) \sin \alpha\right]$

C_h elevator hinge-moment coefficient

$R = (V_s/V)^2$

$T_c' = \dfrac{T}{\frac{1}{2}\rho S_w V^2}$

$T_c'_{u'} = \dfrac{dT_c'}{du'}$

$k_1 = \dfrac{k}{c_e}\dfrac{S_w}{S_e}$

$R_Y = k_Y/C$

$u' = \Delta V/V$

$C_{D_1} = \dfrac{W}{\frac{1}{2}\rho S_w V_o^2} \sin \gamma_o$

L-399

$$C_{L_i} = \frac{W}{\frac{1}{2}\rho S_w V_o^2} \cos \gamma_o$$

$$C_{m_{u'}} = \frac{\partial C_m}{\partial u'}$$

$$C_{M_{u'}} = \frac{\partial C_M}{\partial u'}$$

$$C_{m_\alpha} = \frac{\partial C_m}{\partial \alpha}$$

$$C_{m_\delta} = \frac{\partial C_m}{\partial \delta_e}$$

$$C_{h_{\delta_e}} = \frac{\partial C_h}{\partial \delta_e}$$

$$C_{h_{\alpha_t}} = \frac{\partial C_h}{\partial \alpha_t}$$

$$C_{L_\alpha} = \frac{dC_L}{d\alpha} \text{ per radian}$$

$$C_{D_\alpha} = \frac{dC_D}{d\alpha} \text{ per radian}$$

$$f = \frac{\text{stick force}}{\frac{1}{2}\rho S_w V_o^2}$$

$$\mu = \frac{m}{\rho S_w c}$$

$$K_1 = \frac{R_Y^2 \rho V_o^2}{8\mu\left(\frac{W}{S_w}\right)}$$

Subscripts:

o equilibrium condition

nt tail removed

THE DETERMINATION OF LONGITUDINAL STABILITY CHARACTERISTICS FROM WIND-TUNNEL TESTS OF A MODEL EQUIPPED WITH RUNNING PROPELLERS

A significance of wind-tunnel data obtained with running propellers for the determination of longitudinal-stability characteristics can be evaluated by comparison with existing criterions for longitudinal stability commonly used in flight-test work. One of the simpler experiments to perform in flight consists in determining the position of the elevator required for trim over a range of flying speeds with the throttle setting fixed. When the elevator positions thus obtained are plotted against lift coefficient (or airspeed), an index of the stick-fixed stability, which is commonly used in flight work, results. This index of stability is the slope $(d\delta_e/dC_L)_{trim}$ [or $(d\delta_e/dV)_{trim}$]. This criterion of stability is also associated with the slope of the resultant pitching-moment curve - a curve readily obtained from wind-tunnel data. It is of interest then to know specific quantitative relationships involving these quantities in power-on flight. These relationships are subsequently developed.

The reader who wishes to acquaint himself with the desired fundamental relationships without familiarizing himself with the details of proof may omit the following development and turn immediately to equation (10).

Theory

The equations of equilibrium for an airplane in power-on flight subjected to the force system illustrated in figure 1 may be written as follows:

$$T \cos \alpha - C_D \tfrac{1}{2}\rho S_w V^2 - mg \sin \gamma = 0 \tag{1a}$$

$$-T \sin \alpha - C_L \tfrac{1}{2}\rho S_w V^2 + mg \cos \gamma = 0 \tag{1b}$$

$$Th + C_m \tfrac{1}{2}\rho S_w c V^2 = 0 \; (=C_M \tfrac{1}{2}\rho S_w c V^2) \tag{1c}$$

$$\tfrac{1}{2}\rho S_e c_e V^2 C_h = 0 \tag{1d}$$

If it is assumed in equation 1 that the throttle setting is fixed, any small increments imposed on any of the variables must be such as to maintain equilibrium. After a small change in attitude, equation (1) may be transformed into the following set of equations:

$$(2P_D - 2C_D)\frac{\Delta V}{V} + \left[(C_L - C_{L_1}) - C_{D_\alpha}\right]\Delta\alpha - C_{L_1}\Delta\gamma = 0 \qquad (2a)$$

$$(2P_L + 2C_L)\frac{\Delta V}{V} + \left[(C_D + C_{D_1}) + C_{L_\alpha}\right]\Delta\alpha + C_{D_1}\Delta\gamma = 0 \qquad (2b)$$

$$\frac{2\mu}{R_Y{}^2} C_{M_u{}'}\frac{\Delta V}{V} + \frac{2\mu}{R_Y{}^2} C_{m_\alpha}\Delta\alpha + \frac{2\mu}{R_Y{}^2} C_{m_\delta}\Delta\delta_e = 0 \qquad (2c)$$

$$2C_h\frac{\Delta V}{V} + C_{h_\alpha}\Delta\alpha + C_{h_\delta}\Delta\delta_e = k_1 f \qquad (2d)$$

From the foregoing equations the following relationships between the increments δ_e, α, and f may be established:

$$\frac{d\delta_e}{d\alpha} = \frac{\Delta\delta_e}{\Delta\alpha} = -\frac{E}{F} = \frac{k_1 f}{G} \qquad (3)$$

where

$$E = -\frac{4\mu C_{m_\alpha}}{R_Y{}^2}\left[C_{L_1}(C_L + P_L) - C_{D_1}(C_D - P_D)\right] + \frac{2\mu C_{M_u{}'}}{R_Y{}^2}\left[C_{L_1}(C_{L_\alpha} + C_D) + C_{D_1}(C_L - C_{D_\alpha})\right]$$

$$F = \frac{4\mu C_{m_\delta}}{R_Y{}^2}\left[C_L(C_L + P_L) - C_{D_1}(C_D - P_D)\right]$$

$$G = -\frac{4\mu}{R_Y{}^2}(C_{h_\delta}C_{m_\alpha} - C_{h_\alpha}C_{m_\delta})\left[C_{L_1}(C_L + P_L) - C_{D_1}(C_D - P_D)\right]$$

$$+ \frac{2\mu}{R_Y{}^2} C_{h_\delta}(C_{M_u{}'} - 2C_{m_\delta})\left[C_{L_1}(C_L + P_L) - C_{D_1}(C_D - P_D)\right]$$

The term E is the familiar constant term associated with the biquadratic equation for power-on stick-fixed stability. If the airplane is suddenly displaced from its equilibrium condition, it will continue to diverge from this position if E is negative. Hence, for stability, E must remain positive. If the substitutions

$$C_{L_1} = \frac{W}{\frac{1}{2}\rho S_w V_o^2} (\cos \gamma_o)$$

and

$$C_{D_1} = \frac{W}{\frac{1}{2}\rho S_w V_o^2} (\sin \gamma_o)$$

are made, E may be rewritten as

$$E = -\frac{4\mu}{R_Y^2} \frac{W}{S_w} \frac{2}{\rho V_o^2} \left\{ C_{m_\alpha} \left[\cos \gamma_o (C_L + P_L) - \sin \gamma_o (C_D - P_D) \right] - \frac{1}{2} C_{M_u'} \left[\cos \gamma_o \left(C_{L_\alpha} + C_D \right) + \sin \gamma_o \left(C_D + C_{D_\alpha} \right) \right] \right\} \quad (4)$$

Equation (4) is the general expression for E for power-on flight. For small values of γ_o, $\sin \gamma_o (C_D - P_D) << \cos \gamma_o (C_L + P_L)$ and $\sin \gamma_o (C_D + C_{D_\alpha}) << \cos \gamma_o (C_{L_\alpha} + C_D)$; also, $P_L << C_L$, $C_D << C_{L_\alpha}$, and $\cos \gamma_o = 1$. With these simplifications, equation (4) reduces to

$$E = -\frac{4\mu}{R_Y^2} \frac{W}{S_w} \left(C_{m_\alpha} C_L - \frac{C_{M_u'}}{2} C_{L_\alpha} \right) \left(\frac{2}{\rho V_o^2} \right)$$

or

$$\left(\frac{K_1}{C_L}\right) E = \left(C_{m_\alpha} - \frac{1}{2}\,\frac{C_{L\alpha}}{C_L}\,C_{M_{u'}}\right) \tag{4a}$$

L-539

where

$$K_1 = -\frac{R_Y{}^2 \rho V_o{}^2}{8\mu \frac{W}{S_w}}$$

Further, if the notation is changed

$$\left(\frac{K_1}{C_L}\right) E = \left(\frac{\partial C_m}{\partial \alpha}\right) - \frac{1}{2}\,\frac{dC_L}{d\alpha}\,\frac{1}{C_L}\left(V\,\frac{\partial C_m}{\partial V} + V\,\frac{dT_c'}{dV}\,\frac{h}{c}\right)$$

dividing by $dC_L/d\alpha$ yields

$$\frac{K_1}{C_L}\,\frac{d\alpha}{dC_L}\,E = \frac{\partial C_m}{\partial C_L} - \frac{1}{2C_L}\left(V\,\frac{\partial C_m}{\partial V} + V\,\frac{dT_c'}{dV}\,\frac{h}{c}\right) \tag{5}$$

Now, from equation (1c)

$$C_M = C_m + T_c'\,\frac{h}{c} = 0$$

and on differentiation

$$\left(\frac{dC_M}{dC_L}\right)_{C_M=0} = \frac{dC_m}{dC_L} + \frac{dT_c'}{dC_L}\,\frac{h}{c} \tag{6}$$

For the small values of θ

$$W = \frac{1}{2}\,\rho S_w V^2 C_L$$

or

$$V^2 C_L = \text{constant (approx.)}$$

Hence

$$2VC_L \, dV + V^2 \, dC_L = 0$$

and

$$\frac{dV}{dC_L} = -\frac{V}{2C_L}$$

Under the conditions $V^2 C_L$ = constant and constant throttle operation

$$C_m = C_m(C_L, V)$$

or

$$\frac{dC_m}{dC_L} = \left(\frac{\partial C_m}{\partial C_L}\right)_V + \left(\frac{\partial C_m}{\partial V}\right)_{C_L} \left(\frac{dV}{dC_L}\right)_{V^2 C_L = \text{constant and constant throttle}}$$

Further

$$\frac{dC_m}{dC_L} = \frac{\partial C_m}{\partial C_L} - \frac{V}{2C_L}\left(\frac{\partial C_m}{\partial V}\right) \tag{6a}$$

and

$$\frac{dT_c'}{dC_L} = \frac{dT_c'}{dV}\,\frac{dV}{dC_L} = -\frac{V}{2C_L}\left(\frac{dT_c'}{dV}\right) \tag{6b}$$

The substitution of equations (6a) and (6b) in equation (6) gives

$$\left(\frac{dC_m}{dC_L}\right)_{C_L=0} = \left(\frac{\partial C_m}{\partial C_L}\right) - \frac{1}{2C_L}\left[V\left(\frac{\partial C_m}{\partial V}\right) + V\left(\frac{dT_c'}{dV}\right)\left(\frac{h}{c}\right)\right] \tag{7}$$

and now equation (5) may be rewritten as

L-309

$$\frac{K_1}{C_L} \frac{1}{\frac{dC_L}{d\alpha}} E = \left(\frac{dC_M}{dC_L}\right)_{C_M=0} \tag{8}$$

Equation (7) establishes the relation between the constant term E of the stability biquadratic for power-on flight and the slope of the resultant pitching-moment curve for power-on flight. It should be noted, however, that in equations (6) and (7), the terms C_L and dC_m/dC_L must include any effects of the slipstream; that is,

$$C_L = \left(C_L\right)_{T_c'=0} + \left(\Delta C_L\right)_{slipstream} \tag{9a}$$

and

$$\frac{dC_L}{d\alpha} = \left(\frac{\partial C_L}{\partial \alpha}\right)_{T_c'=0} + \left(\frac{\partial C_L}{\partial V}\right)_\alpha \frac{dV}{dT_c'} \left(\frac{dT_c'}{d\alpha}\right)_{V^2 C_L = \text{constant and constant throttle}} \tag{9b}$$

The relationship $d\delta_e/d\alpha = -E/F$ can now be developed in terms of the quantity $\left(\frac{dC_M}{dC_L}\right)_{C_M=0}$

From equation (3)

$$\left(\frac{d\delta_e}{d\alpha}\right)_{C_M=0} = -\frac{E}{F} = -\left\{\frac{\frac{W}{S_w} \frac{2}{\rho V_o^2} C_L C_{L_\alpha} \left(\frac{dC_M}{dC_L}\right)_{C_M=0}}{C_{m_\delta}\left[C_{L_1}\left(C_L + P_L\right) - C_{D_1}\left(C_D - P_D\right)\right]}\right\}$$

Now

$$C_{L_1}(C_L + P_L) - C_{D_1}(C_D - P_D) = \frac{W}{S_w} \frac{2}{\rho V_o^2}\left(\frac{W}{S_w} \frac{2}{\rho V_o^2} - C_L \frac{dT_c'}{dC_L} \sin\theta_o\right)$$

$$= \left(\frac{W}{S_w} \frac{2}{\rho V_o^2}\right)^2 = C_L^2$$

for small values of θ. Hence

$$\left(\frac{d\delta_e}{d\alpha}\right)_{C_M=0} = \frac{C_L{}^2 C_{L_\alpha}\left(\frac{dC_M}{dC_L}\right)_{C_M=0}}{-\,C_{m_\delta}\;C_L{}^2} = \frac{C_{L_\alpha}\left(\frac{dC_M}{dC_L}\right)_{C_M=0}}{-C_{m_\delta}}$$

and

$$\left(\frac{d\delta_e}{dC_L}\right)_{C_M=0} = \frac{\left(\frac{dC_M}{dC_L}\right)_{C_M=0}}{-\,\frac{\partial C_m}{\partial \delta_e}} \tag{10}$$

Equation (10) states precisely that for power-on flight with a fixed throttle setting, the flight index of stick-fixed stability $\left(\frac{d\delta_e}{dC_L}\right)_{C_M=0}$ is proportional to the slope of the resultant pitching-moment curve at only the point $C_M = 0$. The proportionality factor is the negative reciprocal of the elevator effectiveness parameter, C_{m_δ}.

In equation (10) the slope dC_M/dC_L may be evaluated from the wind-tunnel test data. The slope of the wind-tunnel resultant pitching-moment curve, however, depends on the test procedure adopted for the investigation. In the present analysis two procedures for testing a model equipped with running propellers will be considered. In one method, the model propeller thrust is held constant as the angle of attack is varied, the process being repeated for different amounts of thrust. The expression "constant thrust" will be associated with this test procedure. In the other method, the thrust is varied with lift coefficient in a predetermined manner such as to simulate the thrust condition that exists on the full-scale airplane when flown at a fixed manifold pressure (constant throttle setting). (For constant-speed propeller operation the propeller speed is also constant.) This type of testing will be referred to as the "constant-power" procedure. The constant-thrust procedure will be considered first.

L-309

Constant-Thrust Procedure

For the purposes of analysis, an airplane of the single-engine tractor type with conventional tail arrangement will be considered. If the entire tail surface is assumed to be subjected to slipstream action, the resultant pitching-moment coefficient may be written as follows:

$$C_M = C_{m_o} + (C_g - C_a)\,C_L - \left(\frac{z}{c}\right) C_{X_W} - RC_{L_t}\,\frac{S_t}{S_W}\,\frac{l_1}{c}$$

$$+ \frac{l_2}{c}\,\frac{8}{\pi}\,\frac{S_p}{S_W}\,\frac{K}{J^2}\,\alpha + \frac{h}{c}\,T_c' \qquad (11)$$

The factors C_{m_o}, C_L, and C_{X_W} exclude direct thrust effects but include interference effects due to the slipstream. (See, for example, equation (9a).) When only part of the tail is included in the slipstream, the fourth term on the right side of equation (11) will be lower but will depend on the identical parameters. The fifth term represents the contribution of the aerodynamic side force developed by the inclined propeller (reference 2). In this analysis, this contribution will be grouped with the aerodynamic terms rather than with the direct contribution of the thrust.

If

$$R = 1 + T_c'\,\frac{S_W}{S_p}$$

and it is assumed

$$\frac{dR}{dC_L} = \frac{dR}{dT_c'}\,\frac{dT_c'}{dC_L}$$

differentiation with respect to the over-all lift coefficient yields

$$\frac{dC_M}{dC_L} = (C_g - C_a) - \frac{z}{c}\frac{dC_{X_W}}{dC_L} - R\frac{l_1}{c}\frac{S_t}{S_W}\frac{dC_{L_t}}{dC_L}$$

$$+ \frac{l_2}{c}\frac{8}{\pi}\frac{S_p}{S_W}\frac{K}{J^2}\frac{d\alpha}{dC_L} + \frac{h}{c}\frac{dT_c'}{dC_L} - C_{L_t}\frac{l_1}{c}\frac{S_t}{S_p}\frac{dT_c'}{dC_L} \quad (12)$$

Equation (12) represents the general variation of the resultant pitching-moment coefficient with lift coefficient as it includes terms involving the variation of propeller thrust. If the constant-thrust test procedure is employed (T_c' = constant), equation (12) reduces to

$$\left(\frac{dC_M}{dC_L}\right)_{T_c'} = \left(\frac{\partial C_M}{\partial C_L}\right) = (C_g - C_a) - \frac{z}{c}\frac{dC_{X_W}}{dC_L}$$

$$- R\frac{l_1}{c}\frac{S_t}{S_W}\frac{dC_{L_t}}{dC_L} + \frac{l_2}{c}\frac{8}{\pi}\frac{S_p}{S_W}\frac{K}{J^2}\frac{d\alpha}{dC_L} = \frac{\partial C_m}{\partial C_L} \quad (12a)$$

The term dC_{L_t}/dC_L is assumed to be independent of T_c' because

$$\frac{dC_{L_t}}{dC_L} = \frac{dC_{L_t}}{d\alpha_t}\left(\frac{d\alpha}{dC_L} - \frac{d\epsilon_W}{dC_L} - \frac{d\epsilon_p}{dC_L}\right) .$$

and experiments indicate that $d\epsilon_p/dC_L$ is essentially constant, at least for the flap-up condition. It is seen that the value of $\left(\frac{dC_M}{dC_L}\right)_{T_c'}$ varies with the magnitude of T_c' but that it is essentially independent of the tail load and hence of the value of the pitching moment at which the slope is measured. Accordingly, it makes little difference whether the model is trimmed or not and the slope $\left(\frac{dC_M}{dC_L}\right)_{T_c'}$ may be evaluated with any tail setting, although

L-309

it should be appreciated that the slope of the pitching-moment curve obtained from the constant-thrust procedure is, by itself, meaningless from the consideration of stability in steady flight.

In order to estimate the stability parameter $\left(\frac{dC_M}{dC_L}\right)_{C_M=0}$ from wind-tunnel data obtained by the constant-thrust procedure, it is now necessary to evaluate the terms

$$\frac{1}{2C_L}\left[V\left(\frac{\partial C_m}{\partial V}\right) + V\frac{dT_c'}{dV}\frac{h}{c}\right] = -C_{L_t}\frac{l_1}{c}\frac{S_t}{S_p}\frac{dT_c'}{dC_L} + \frac{h}{c}\frac{dT_c'}{dC_L}$$

where

$$C_{L_t} = -\frac{\left(C_{M_{nt}}\right)_{T_c'}}{\left(1 + T_c'\frac{S_w}{S_p}\right)\frac{S_t}{S_w}\frac{l_1}{c}}$$

Thus

$$\left(\frac{dC_M}{dC_L}\right)_{C_M=0} = \left(\frac{\partial C_M}{\partial C_L}\right)_{T_c'} + \frac{\left(C_{M_{nt}}\right)_{T_c'}}{\left(1 + T_c'\frac{S_w}{S_p}\right)\frac{S_t}{S_w}\frac{l_1}{c}}\frac{l_1}{c}\frac{S_t}{S_p}\frac{dT_c'}{dC_L} + \frac{h}{c}\frac{dT_c'}{dC_L}$$

$$= \left(\frac{\partial C_M}{\partial C_L}\right)_{T_c'} + \frac{dT_c'}{dC_L}\left[\frac{\left(C_{M_{nt}}\right)_{T_c'}}{\left(1 + T_c'\frac{S_w}{S_p}\right)\frac{S_t}{S_w}\frac{l_1}{c}}\frac{l_1}{c}\frac{S_t}{S_p} + \left(\frac{h}{c}\right)\right] \quad (13)$$

The foregoing relationships can be used to estimate $\left(\frac{dC_M}{dC_L}\right)_{C_M=0}$ when tunnel data are available in the form $\left(\frac{dC_M}{dC_L}\right)_{T_c'}$ for

both tail-on and tail-off conditions and when propeller data are available to evaluate the quantity, dT_c'/dC_L.

From equation (6b)

$$\frac{dT_c'}{dC_L} = -\frac{V}{2C_L}\left(\frac{dT_c'}{dV}\right)$$

The slope dT_c'/dV is best determined graphically for the particular condition under consideration. A method for finding the variation of propeller thrust with forward velocity for either fixed-pitch or controllable-pitch constant-speed propellers is outlined in reference 3. Figure 2 typifies the variation of thrust coefficient and blade angle with velocity for constant-speed propeller operation.

Constant-Power Procedure

If in equation (12) the thrust is varied in accordance with the relation $\frac{dT_c'}{dC_L} = -\frac{V}{2C_L}\left(\frac{dT_c'}{dV}\right)$, equation (12) is representative of the constant-power test procedure. The direct relationship between the resultant slope of the pitching-moment curve for the constant-power test procedure and the constant term E of the stability biquadratic for power-on flight has already been established.

It is obvious from equation (12) that, in the constant-power test procedure, the tail setting directly affects the measured wind-tunnel pitching-moment slope, dC_M/dC_L. In evaluating dC_M/dC_L it is consequently important to use that tail setting for which the model is trimmed (that is, $C_M = 0$). It will be observed from equation (12) that, in power-on flight, the slope of the resultant pitching-moment curve (and consequently the stability characteristics) is affected by both the center-of-gravity location and the associated tail load necessary to produce trim. The manner in which these associated variables affect the stability characteristics forms the subject matter for the remainder of this paper.

L-309

EFFECT OF TAIL LOAD AND CENTER-OF-GRAVITY POSITION ON THE SLOPE OF THE PITCHING-MOMENT CURVE

In general, the contribution of the tail load to the resultant pitching-moment slope, dC_M/dC_L, is expressed by final term in equation (12). Thus

$$\Delta\left(\frac{dC_M}{dC_L}\right)_{tail} = - C_{L_t} \frac{l_1}{c} \frac{S_t}{S_p} \frac{dT_c'}{dC_L}$$

where

$$C_{L_t} = \left(\frac{dC_{L_t}}{d\alpha_t}\right)\left[\alpha - \epsilon_w - \epsilon_p + \left(\frac{\partial \alpha_t}{\partial \delta_e}\right) \delta_e + i_t\right]$$

The effect of this term on dC_M/dC_L is demonstrated in figure 3. The theoretical curve was evaluated by use of the propeller-operating characteristics that were used in securing the experimental results. The experimental points were obtained from data of unpublished tests. Both the theoretical and the experimental results indicate that increased down loads on the tail result in more positive values of dC_M/dC_L and thus are detrimental to stability.

In accordance with equation (11), the downward tail load must be increased to preserve the trim condition when the center of gravity of the airplane is moved forward. In power-off flight, a forward movement of the center of gravity is normally stabilizing, as it results in more negative values of dC_M/dC_L. It has just been shown, however, that an increasing down load on the tail is detrimental to stability in power-on flight. Thus, the two effects oppose one another. The results of a theoretical examination of these antithetical effects of a center-of-gravity movement are presented in figures 4 and 5. In the computations for figure 4, the elevator angles necessary to trim the airplane with the various center-of-gravity positions were computed and the associated values of dC_M/dC_L and $\left(\frac{\partial C_M}{\partial C_L}\right)_{T_c'}$ were calculated. The value of dC_M/dC_L is

the slope of the pitching-moment curve associated with constant-power tests: $\left(\frac{\partial C_M}{\partial C_L}\right)_{T_c'}$ is the slope of the pitching-moment curve associated with constant-thrust tests. It will be noted that the variations of both dC_M/dC_L and $\left(\frac{\partial C_M}{\partial C_L}\right)_{T_c'}$ with forward center-of-gravity movement indicate a net increase in stability, but that the increased negative values of dC_M/dC_L are less than the increased negative values of $\left(\frac{\partial C_M}{\partial C_L}\right)_{T_c'}$. A comparison of these two quantities reveals directly the effect of the increased down load on the tail required for trim with the forward center-of-gravity position, for $\left(\frac{\partial C_M}{\partial C_L}\right)_{T_c'}$ includes the effect of the shift in center-of-gravity location but not the change in tail load, whereas dC_M/dC_L includes both of these variations.

Computations that show the variations of dC_M/dC_L, $\left(\frac{\partial C_M}{\partial C_L}\right)_{T_c'}$, and the elevator angle for trim with lift coefficient for two center-of-gravity positions are presented in figure 5. The variation of the thrust coefficient, T_c', with lift coefficient is also shown. The thrust coefficients were used in evaluating $\left(\frac{\partial C_M}{\partial C_L}\right)_{T_c'}$. The more negative values of dC_M/dC_L and the steeper slope to the elevator angle for trim curve are associated with the most forward center-of-gravity location. It will be noted that for values of C_L greater than 1, the slope dC_M/dC_L for the 26-percent mean aerodynamic chord center-of-gravity position is greater than the associated slope $\left(\frac{\partial C_M}{\partial C_L}\right)_{T_c'}$. This behavior results from the increased positive (upward) tail loads required for trim at the higher lift coefficients. Thus, it is seen that the effect of power on the contribution of the tail to the stability characteristics is adverse only when the tail is carrying an initial down load.

CONCLUSIONS

On the basis of this analysis, the following conclusions may be reached:

1. For small angles of climb, the slope of the curve of elevator angle for trim against lift coefficient secured from flight tests is directly proportional to the slope of the curve of pitching-moment coefficient against lift coefficitnt secured from wind-tunnel tests simulating flight with constant power only when the model is trimmed for zero pitching-moment.

2. The destabilizing effects of power are more pronounced when the horizontal tail is required to carry a down load to maintain flight equilibrium.

3. A longitudinal movement of the center of gravity affects the longitudinal stability characteristics less in power-on flight than in power-off flight.

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L-309

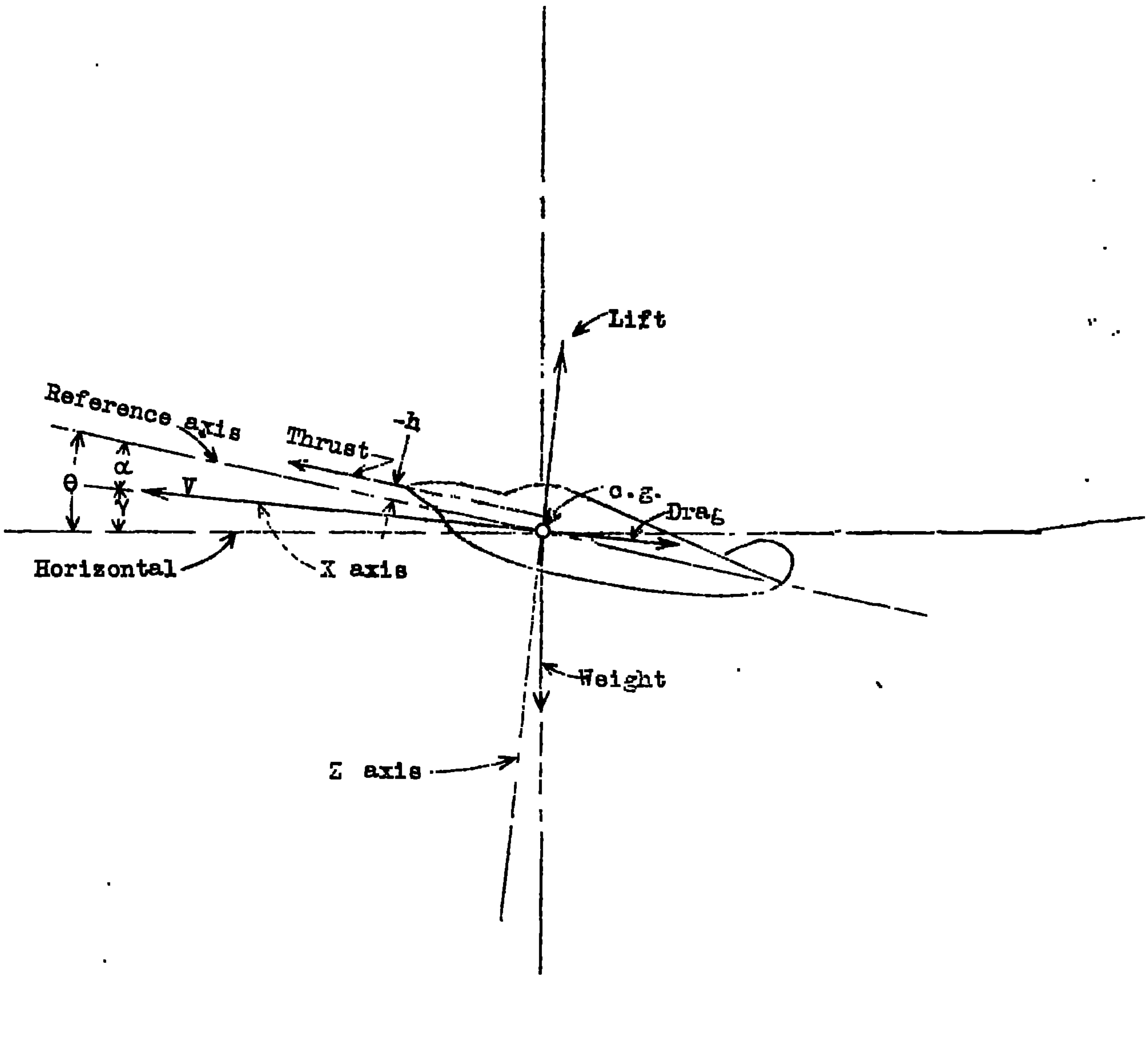

Figure 1.- Angular and vectorial relationships in power-on flight. Flight-path axis.

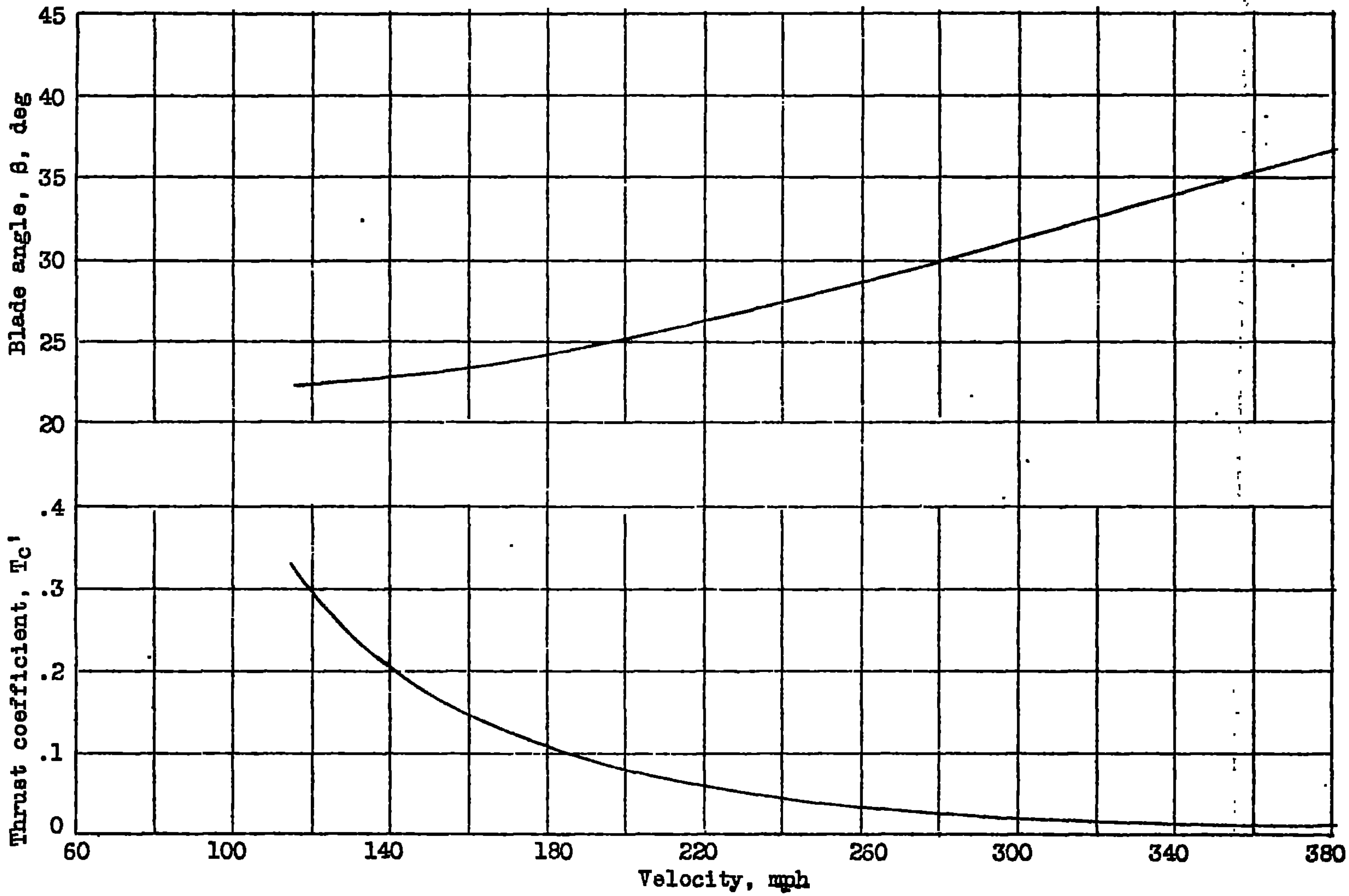

Figure 2.- Typical variation of thrust coefficient and blade angle with velocity for constant-speed propeller.

L-509

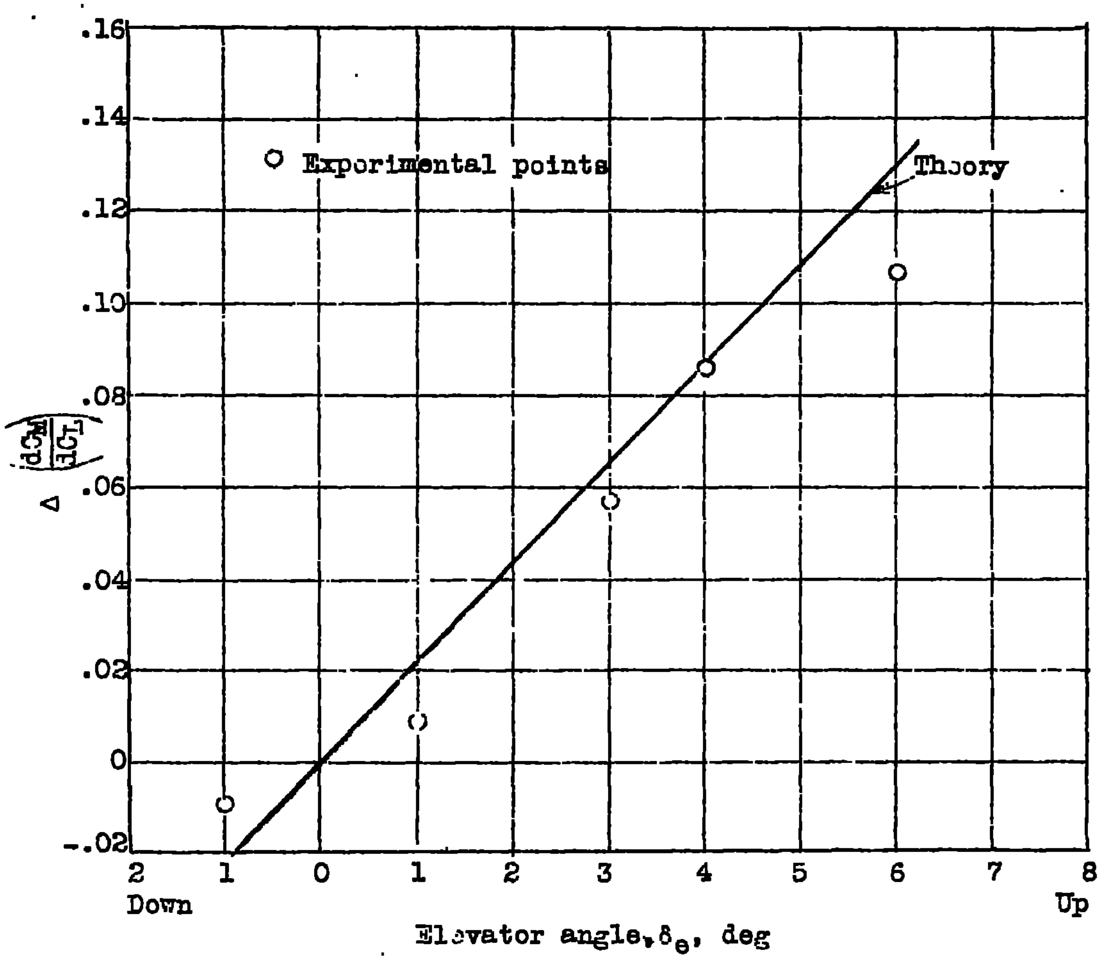

Figure 3.- Effect of tail load on slope of resultant pitching-moment curve. $T_c{}'$, 0.195, C_L, 0.6.

L-309

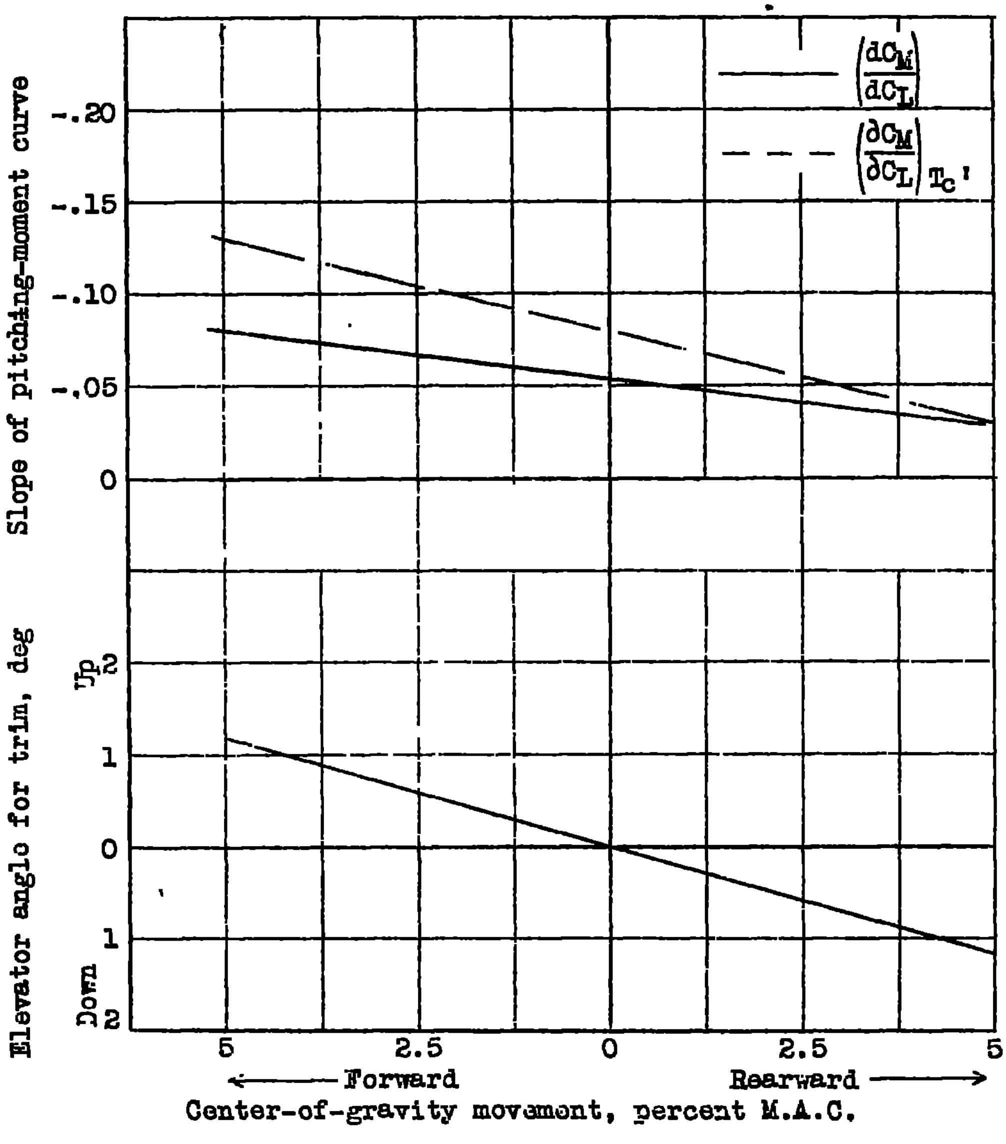

Figure 4.- Effect of center-of-gravity movement on slope of pitching-moment curve and on elevator angle for trim. T_c', 0.195, C_L, 0.5.